1997 SGHS

QUESTION 1

a) Integrate ;

i) $I = \int \frac{e^x}{2e^x + 1}\,dx$

i) $I = \int \sin^2 4x\,dx$

b) By using the substitution $u = 1 - r^3$ integrate $I = \int \frac{9r^2\,dr}{\sqrt{1 - r^3}}$

c) Evaluate $\int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{9 + x^2}$

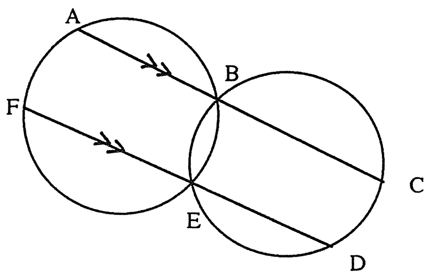
d) If $y = 2\sin^{-1}\left(\frac{x-1}{2}\right)$

i) Write down the domain and range

i) Find $\frac{dy}{dx}$

iii) Find the equation of the tangent at $x = 1$

QUESTION 2



a) Given A C parallel to FD, prove ACDF is a parallelogram.

b) Solve the inequality and graph the solution on a number line ; $\frac{2x+3}{x-1} > 1$

c) Prove by Mathematical Induction for positive integral values of n ;

$1 + 4 + 4^2 + 4^3 + \ldots\ldots\ldots\ldots + 4^{n-1} = \frac{4^n - 1}{3}$

d) Evaluate ; $I = \int_0^{\frac{3}{4}} \frac{dx}{\sqrt{9 - 16x^2}}$

QUESTION 3

a) Solve the equation $2\cos^2 x + 3\sin x - 3 = 0$ for $0 \le x \le 2\pi$

b) By expressing $\sqrt{3}\cos x - \sin x$ in the form $A\cos(x + \alpha)$ where α is acute , solve the equation $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$

c) i) Find the value of k for which the polynomial $9x^4 - 25x^2 + 10kx - k^2$ is divisible by both $(x - 1)$ and $(x + 2)$.

ii) With this value of k , find the roots of the equation $9x^4 - 25x^2 + 10kx - k^2 = 0$

d) i) Show that the polynomial $p(x) = x^3 + x - 1$ has a root in the interval $0 < x < 1$

ii) Starting with the first approximation that $x_1 = 0.5$, use Newton's Method for the above polynomial once to find a better approximation correct to 3 sig. figs.

QUESTION 4

a) In a certain municipality the ratio of boy births to girl births is 11 : 10. A certain couple in the community wish to have at least one girl in their family. How many children must they have to be 90% confident of having at least one girl in the family ?

b) Let A(-4, -7) and B(2, 2) be points in the number plane. Find the co ordinates of the point P (x, y) which divides the interval externally in the ratio 4 : 1.

c) Find the minimum value of $y = 4\cos 4x$ for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$

d) A projectile is fired with velocity 900 *m/s* at an angle α such that $\tan\alpha = \frac{4}{3}$, assume $g = 10 m/s^2$.

i) Derive the horizontal and vertical equations of motion and position for the projectile.

ii) Find the maximum height reached by the projectile

iii) Find the maximum distance reached by the projectile.

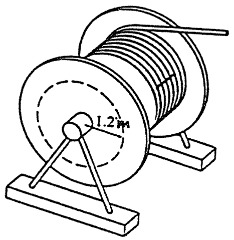
iv) If the angle of projection (ie α) was increased by 10% , would there be corresponding increase in the distance reached (range) by the projectile ? Justify your answer.

QUESTION 5

a) A man walks 500 metres from A to B on a flat, straight road. From A a tower is directly North with an angle of elevation to its top of 37^0. From B the tower is due west and the angle of elevation to the top is 30^0. Find the height of the tower to the nearest metre.

b) A lady has three bags of fruit. Bag A contains 5 red and 3 green apples, Bag B contains 4 red and 3 green apples and Bag C contains 3 red and 7 green apples. One bag is selected at random and from the bag two apples are selected without replacement. What is the probability that the two apples selected are both red ?

c)



As an Optus television cable is pulled from a large spool to be strung from the telephone poles along the street, it unwinds from the spool in layers of constant radius (see figure). If the truck pulling the cable moves at a steady 6m / sec, use the equation $s = r\theta$ to find out how fast (radians /sec) the spool is turning when the layer of radius 1.2m is being unwound.

d) The acceleration of a particle travelling in a straight line is given by $a = \frac{1}{(t+1)^2}$ where t is in seconds. If the body starts at rest from the origin find ;

i) An expression for the velocity.
ii) The velocity after 4 seconds.
iii) An expression for the displacement.
iv) Find the displacement after 1 second.
v) Graph velocity (v) against time.
vi) Graph position (x) against time.

QUESTION 6

a) Show that $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

b) Given a particle oscillates in Simple Harmonic Motion such that $\ddot{x} = -n^2x$, express v as a function of x. The amplitude of the motion is a.

c) A particle experiencing Simple Harmonic Motion has a velocity of 8 cm/sec towards O when it is 1.5cm from O. If the period of the motion is $\frac{\pi}{2}$ seconds, find ;

i) the maximum velocity of the particle.
ii) the maximum displacement of the particle.
iv) the maximum acceleration of the particle.

d) i) Differentiate $y = \tan^{-1}\frac{1}{x}$ $x \neq 0$

and hence show $\frac{d}{dx}\left(\tan^{-1}x + \tan^{-1}\frac{1}{x}\right) = 0$

ii) Sketch the curve $y = \tan^{-1}x + \tan^{-1}\frac{1}{x}$

QUESTION 7

a) Points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. If the chord PQ subtends an angle of 90^0 at the vertex of the parabola, prove that PQ cannot be a focal chord.

b) Assume that the rate at which a body warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air. This rate can be expressed by the differential equation $\frac{dT}{dt} = k(T - A)$ where t is the time in minutes and k is a constant.

i) Show that $T = A + Ce^{kt}$, (where C is a constant) is a solution to the differential equation.

ii) A cooled body warms from 5^oC to 15^oC in thirty minutes. The air temperature is 40^oC ;

α) Find the temperature of the body after a further 30 minutes (ans to the nearest degree)

β) Find how long (in minutes) the body will take to reach a temperature of 30^oC

δ) Explain what will happen to T as t becomes large.

Question One 1997 SGHS (3U T 1)

a) (i) $I = \int \frac{e^x}{2e^x+1}\,dx = \frac{1}{2}\int \frac{2e^x}{2e^x+1} = \frac{1}{2}\ln(2e^x+1) + c$

(ii) $I = \int \sin^2 4x \, dx = \int \frac{1}{2}(1-\cos 8x)\,dx$

$= \frac{1}{2}x - \frac{1}{16}\sin 8x + c$

b) $I = \int \frac{9r^2\,dr}{\sqrt{1-r^3}}$ let $u = 1-r^3$

$\frac{du}{dr} = -3r^2 \Rightarrow du = -3r^2\,dr$

$I = -3\int \frac{du}{\sqrt{u}} = -3\int u^{-\frac{1}{2}}\,du$

$= -6\sqrt{1-r^3} + c$

c) $I = \int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{9+x^2} = \left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_{\sqrt{3}}^{3\sqrt{3}}$

$= \frac{1}{3}\left[\tan^{-1}(\sqrt{3}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

$= \frac{1}{3}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{\pi}{18}$

) $y = 2\sin^{-1}\left(\frac{x-1}{2}\right)$ (i) $D_x: \{-1 \le \frac{x-1}{2} \le +1\}$

$\{-1 \le x \le 3\}$

$R_y: \{-\pi \le y \le +\pi\}$

(ii) $y = 2\sin^{-1}\left(\frac{x-1}{2}\right)$

$\frac{dy}{dx} = 2 \times \frac{1}{\sqrt{1-\left(\frac{x-1}{2}\right)^2}} \times \frac{1}{2}$

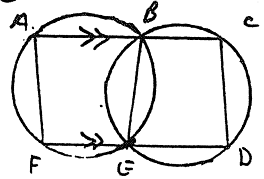
$= \frac{2}{\sqrt{4-(x-1)^2}}$

(iii) at $x = 2$ $y = \frac{\pi}{6}$ $\frac{dy}{dx} = \frac{2}{\sqrt{4-(1)^2}} = \frac{2}{\sqrt{3}}$

Eqn of tangent $y - \frac{\pi}{6} = \frac{2}{\sqrt{3}}(x-2)$

$y = \frac{2}{\sqrt{3}}x + \left(\frac{\pi}{6} - \frac{4}{\sqrt{3}}\right)$

Q2 (a)



Constructions: Join AF, BE, CD.

Now $\angle BAC + \angle BEF = 180°$ (opp $\angle$s of cyclic quad)

and $\angle BED + \angle BEF = 180°$ (supp $\angle$'s)

$\therefore \angle BAC = \angle BED$

But $\angle BED + \angle BCD = 180°$ (opp $\angle$'s of cyclic quad)

i.e. $\angle BAC + \angle BCD = 180°$.

But $\angle BAC$ and $\angle BCD$ are co-interior angles.

$\therefore AF \parallel CD$ (co-interior $\angle$'s are supp for parallel lines)

and $AC \parallel DF$ (data)

$\therefore ACDF$ is a parm (opp sides are parallel)

b) $\frac{2x+3}{x-1} > 1$

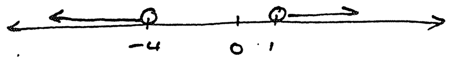
$(x-1)^2 \frac{2x+3}{x-1} > 1.(x-1)^2 \Rightarrow (x-1)(2x+3) - (x-1)^2 > 0$

$(x-1)(\{2x+3\} - \{x-1\}) > 0$

$(x-1)(x+4) > 0$.

$\therefore (x-1) > 0$ $(x+4) < 0$

$x > 1$ $x < -4$



© $1 + 4 + 4^2 + 4^3 .. + 4^{n-1} = \frac{4^n - 1}{3}$.

Step 1: Test for $n = 1$.

L.H.S. = 1 R.H.S. = $\frac{4^1 - 1}{3} = 1$.

$\therefore$ true for $n = 1$.

Step 2: Assume true for $n = k$.

i.e. $1 + 4 + 4^2 + . . + 4^{k-1} = \frac{4^k - 1}{3}$

Prove true for $n = k+1$

Now L.H.S. $1 + 4 + 4^2 + . . + 4^{k-1} + 4^k$ R.H.S $= \frac{4^{k+1} - 1}{3}$.

$= \frac{4^k - 1}{3} + 4k$ (from assumption)

$= \frac{4^k - 1 + 3.4k}{3} = \frac{1.4^k + 3.4^k - 1}{3}$

$= \frac{4.4^k - 1}{3} = \frac{4^{k+1}}{3}$ = R.H.S.

$\therefore$ true for $n = k+1$

Step 3: Now shown true for $n = k+1$, if true for $n = k$,

But true for $n = 1$, so true for $n = 1+1 = 2$, and $n = 2+1 = 3$,

and true for all +ve integer n.

$+x-2)(9x^2-9x+2)$

$(x^2+x-2)(3x-1)(3x-2)$

$\therefore$ Roots $x = 1, -2, \frac{1}{3}, \frac{2}{3}$

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d) (i) $P(x) = x^3+x-1$

$P(0) = -1 \qquad P(1) = +1$

$\therefore P(0) < 0 \quad P(1) > 0$

$\therefore P(x) = 0 \quad 0<x<1.$

ie $P(x)$ has a root in interval $0<x<1$.

(ii) $x_2 = x_1 - \dfrac{P(x_1)}{P'(x_1)}$ $\qquad P(x) = x^3+x-1$

$x_2 = 0.5 - \dfrac{-0.375}{1.75}$ $\qquad P'(x) = 3x^2+1$

$P(0.5) = -0.375$

$\therefore x_2 = 0.714$ (3 sig fig). $\qquad P'(0.5) = 1.75$

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e) $f(x) = \cos x$ $\qquad g(x) = a+bx+cx^2$

$f(0) = \cos 0 = 1$ $\qquad g(0) = a+0+0.$

$f(0) = g(0) \Rightarrow a = 1$

$f'(x) = -\sin x$ $\qquad g'(x) = b+2cx.$

$f'(0) = 0$ $\qquad g'(0) = b.$

$\therefore b = 0$ $\qquad f'(0) = g'(0).$

$f''(x) = -\cos x$ $\qquad g''(x) = 2c.$

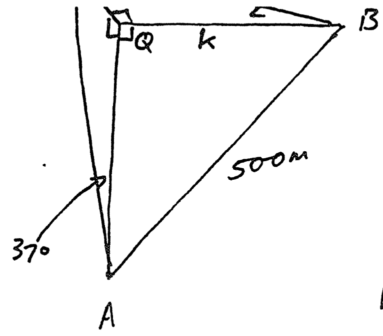
$f''(0) = -1.$ $\qquad g''(0) = 2c$

But $f''(0) = g''(0).$

$\therefore 2c = -1$

$c = -\frac{1}{2}.$

$\therefore a = 1, \; b = 0, \; c = -\frac{1}{2}$



Let $BQ = k$

then $AQ = \sqrt{500^2 - k^2}$

Now: $\tan 30° = \dfrac{h}{k}$. and $\tan 37° = \dfrac{h}{\sqrt{500^2-k^2}}$

$k = \dfrac{h}{\tan 30°}$ $\Rightarrow \tan 37° = \dfrac{h}{\sqrt{500^2 - \frac{h^2}{\tan^2 30}}}.$

$\tan^2 37\left(500^2 - \dfrac{h^2}{\tan^2 30}\right) = h^2$

$\tan^2 37\left(\dfrac{\tan^2 30 . 500^2 - h^2}{\tan^2 30}\right) = h^2$

$\tan^2 37(\tan^2 30).500^2 - \tan^2 37.h^2 = h^2 \tan^2 30.$

$\therefore h^2 = \dfrac{\tan^2 37 . \tan^2 30 . 500^2}{\tan^2 37 + \tan^2 30}$

$h = \dfrac{217.5323168}{0.812120057}$

$h = 267857 \doteqdot 268$ m

(b)

A	B	C
5R 3G	4R 3G	3R 7G.

$P(2R) = P(A)\times P(R)\times P(R) + P(B)\times P(R)\times P(R) + P(C)\times P(R)\times P(R)$

$= \frac{1}{3}\times\frac{5}{8}\times\frac{4}{7} + \frac{1}{3}\times\frac{4}{7}\times\frac{3}{6} + \frac{1}{3}\times\frac{3}{10}\times\frac{2}{9}$

$= \frac{5}{42} + \frac{2}{21} + \frac{1}{45}$

$= \frac{149}{630}.$

(c) Require $\dfrac{d\theta}{dt}$ $\qquad \dfrac{d\theta}{dt} = \dfrac{d\theta}{ds}\cdot\dfrac{ds}{dt}.$ $\qquad s = \theta r.$ $\quad \dfrac{ds}{d\theta} = r.$

$\therefore \dfrac{d\theta}{dt} = \dfrac{1}{r}\cdot 6$ $\qquad \dfrac{ds}{dt} = 6$

$= \dfrac{6}{r}$

when $r = 1.2$

then $\dfrac{d\theta}{dt} = \dfrac{6}{1.2} = 5$ rad/sec

$a = \frac{1}{(t+1)^2} = \frac{d^2x}{dt^2}$ $\quad C=0$ (start at $x=0$)

$+c$

(i) $\frac{d^2x}{dt^2} = (t+1)^{-2} \Rightarrow \frac{dx}{dt} = -(t+1)^{-1} = \frac{-1}{(t+1)}$

(ii) v when $t = 4$ $\quad \frac{dx}{dt} = \frac{-1}{4+1} = -\frac{1}{5}$ m/s

(iii) $\frac{dx}{dt} = \frac{-1}{(t+1)} \Rightarrow x = -\ln(t+1) + c$ at $x=0, t=0$.

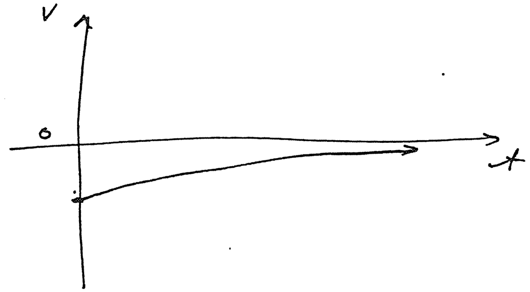
$\therefore c = 0$.

$\therefore x = -\ln(t+1)$

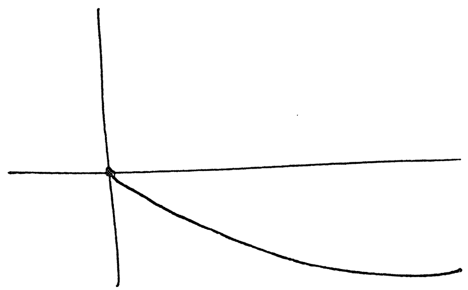
(iv) x when $t=1$ $\quad x = -\ln(1+1)$

$\underline{x = -\ln 2}$

(v)



(vi)



Q5 (a) $B:G = 11:10$

$\therefore 1 - P(\text{All Boys}) \geq 90\%$

$\therefore P(\text{All Boys}) \leq 10\%$

$\left(\frac{11}{21}\right)^n < 0.10 \qquad \underline{\left(\frac{11}{21}\right)^4 < 0.10}$.

$\underline{n < 4}$

$\therefore$ Require 4 children for 90% confidence

(b) $A(-4,-7)$, $B(2,2)$. External $4:1$.

i.e. $4:-1$.

$x = \frac{-4\times -1 + 2\times 4}{4-1} = \frac{4+8}{3} = \frac{12}{3} = 4$.

$y = \frac{-7\times -1 + 2\times 4}{4-1} = \frac{7+8}{3} = \frac{15}{3} = 5$

$\therefore P(4,5)$

(c)(i) Vertical

$\ddot{y} = -g$

$\dot{y} = -gt + c_1$

at $t=0$ $\dot{y} = 900\sin\alpha$.

$\therefore c_1 = 900\sin\alpha$.

$\boxed{\dot{y} = -gt + 900\sin\alpha.}$

$y = -\frac{1}{2}gt^2 + 900t\sin\alpha + c_2$

at $t=0$ $y=0 \Rightarrow c_2 = 0$

$\therefore \boxed{y = -5t^2 + 720t.}$

Horizontal.

$\ddot{x} = 0$

$\dot{x} = c_3$

at $t=0$ $\dot{x} = 900\cos\alpha$.

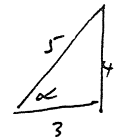
$\therefore c = 900\cos\alpha$.

$\boxed{\dot{x} = 900\cos\alpha.}$

$x = 900t\cos\alpha + c_4$

at $t=0$ $x=0 \Rightarrow c_4 = 0$

$\therefore \boxed{x = 540t}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

(ii) a) Max height occurs $\dot{y} = 0$.

$\therefore 0 = -10t + 720$

$\therefore \underline{t = 72}$ secs.

Max height $= -5(72)^2 + 720\times 72$

$= \underline{25920}$ m.

b) Max distance occurs when $y=0$

$\therefore 5t^2 = 720t$

$t=0$ $t = 144$ secs.

$x = 540t$

$x = 540\times 144$

$x = \underline{77760}$ m.

d) $\alpha = 53°8'$

10% increase $\alpha = 58°27'$ $\quad$ Now $y = -5t^2 + 767t$.

$\therefore t=0,\ t = 153.4$ secs.

Max dist $x = 900t\cos\alpha$

$x = 900\times 153.4\times\cos$ (i.e. actual distance – not ...)

$y = 4\cos 4x$

$\frac{dy}{dx} = -16\sin 4x$

$\frac{dy}{dx} = 0$ for a max/min

$\therefore 16\sin 4x = 0$

$\sin 0 = 0$

$\sin \pi = 0$

$\therefore 4x = \pi$

$x = \frac{\pi}{4}$

Now Test. $y = 4\cos 4\times\frac{\pi}{4}$

$y = 4\cos\pi$

$y = -4$

Test $x = \frac{\pi}{2}$ $\quad y = 4\times\cos 4\times\frac{\pi}{2}$

$y = 4\times\cos 2\pi$

$y = 4\times 1$

$y = +4$

No turning pt in range $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

$\therefore$ Minimum value is -4 at $x = \frac{\pi}{4}$.

4b (a) $\ddot{x} = \frac{d^2x}{dt^2}$

Show $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = v.\frac{dv}{dx}$ $\qquad v = \frac{dx}{dt}$.

$= \frac{dx}{dt}.\frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$= \frac{d^2x}{dt^2}$

$= \ddot{x}$

(b) $\ddot{x} = -n^2x$.

then $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -n^2x$.

$\int\frac{d}{dx}\left(\frac{1}{2}v^2\right).dx = -\int n^2x.dx$

$\frac{1}{2}v^2 = -\frac{n^2}{2}x^2 + c$.

when $x = a$ $v = 0$ $\Rightarrow \therefore \frac{n^2}{2}.a^2 = c$.

Now $\frac{1}{2}v^2 = \frac{-n^2}{2}x^2 + \frac{n^2a^2}{2}$

i.e. $v^2 = n^2(a^2 - x^2)$ $\Big\}$ OR

$v = n.\pm\sqrt{a^2 - x^2}$

(c) Now $v^2 = n^2(a^2 - x^2)$. $\quad v = 8$ when $x = 1.5$. $\quad T = \frac{\pi}{2}$

$T = \frac{2\pi}{n}$.

$\therefore \frac{\pi}{2} = \frac{2\pi}{n}$.

$\therefore 4 = n$

$8^2 = 4^2(a^2 - 1.5^2)$.

$64 = 16(a^2 - 1.5^2)$.

$4 = a^2 - 1.5^2$

$a^2 = 4 + 1.5^2$

$a = 2.5$.

$\therefore v^2 = 4^2(2.5^2 - x^2)$.

(i) Max velocity — at centre of motion $x = 0$

$\therefore v^2 = 4^2(2.5)^2$

$v = |10|$ cm/sec

(ii) Max displacement — i.e. $v = 0$ $\Rightarrow x = 2.5$ (i.e. at amplitude spot).

(iii) Max accelera[illegible] — at end of motion (extremes) $x = 2.5$

$\ddot{x} = -16.x$ $\qquad \ddot{x} = 140$ km/sec^2

$y = \tan^{-1}\left(\frac{1}{x}\right)$

$\frac{d}{dx}(x^{-1}) = -1x^{-2}$

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{1}{x}\right)^2} \times \frac{-1}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{x^2+1}{x^2}} \times \frac{-1}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2}{x^2+1} \times \frac{-1}{x^2} = \frac{-1}{x^2+1}$$

Now if $y = \tan^{-1}(x)$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$\therefore$ If $y = \left(\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)\right)$

then $\frac{dy}{dx} = \frac{1}{1+x^2} \quad \frac{-1}{x^2+1} = 0$

(ii) Sketch. $y = \tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$

Since $\frac{dy}{dx} = 0$ for this curve it is a horizontal line.

does not exist for $x = 0$

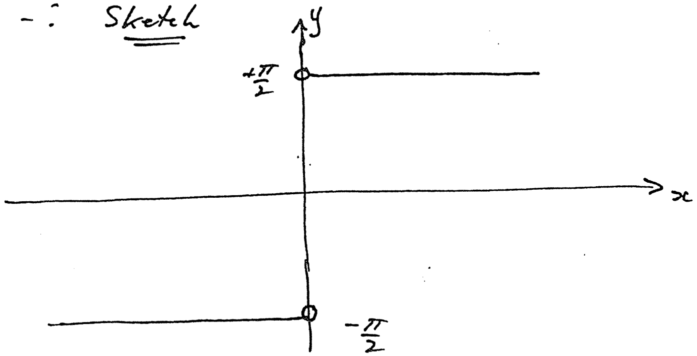
when $x = 1$ $y = \tan^{-1}(1) + \tan^{-1}\left(\frac{1}{1}\right)$

$y = \frac{\pi}{4} + \frac{\pi}{4} \Rightarrow y = \frac{\pi}{2}$.

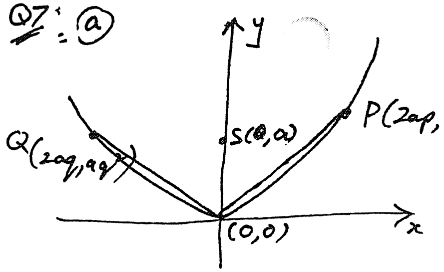
when $x = -1$ $y = \tan^{-1}(-1) + \tan^{-1}\left(\frac{1}{-1}\right)$

$y = -\frac{\pi}{4} + -\frac{\pi}{4} \Rightarrow y = -\frac{\pi}{2}$

$\therefore$ Sketch



Q7: (a)



If PQ subtends an angle of at O.

then grad of PO × grad of OQ = -

Grad of OP $= \frac{ap^2}{2ap} = \frac{p}{2}$.

Grad of OQ $= \frac{aq^2}{2aq} = \frac{q}{2}$.

Now $\frac{p}{2} \times \frac{q}{2} = -1$

$\therefore \boxed{pq = -4}$

Eqn of chord PQ $\frac{y-ap^2}{x-2ap} = \frac{aq^2-ap^2}{2aq-2ap} = \frac{\cancel{a}\cancel{(q-p)}(q+p)}{2\cancel{a}\cancel{(q-p)}}$

$2(y-ap^2) = (q+p)(x-2ap)$.

To be a focal chord PQ must pass thru $(0, a)$

Now $2(a-ap^2) = (q+p)(-2ap)$

$2a - \cancel{2ap^2} = -\cancel{2ap^2} - 2aqp$

$\therefore 2a = -2aqp$.

or $\boxed{pq = -1}$

But from above $pq = -4$

$\therefore$ PQ cannot be a focal chord.

7 marks

(max 4 if $pq = -1$ is not proven)

i) Given $T = A + ce^{kt}$ $\Rightarrow ce^{kt} = T - A$.

$$\frac{dT}{dt} = kce^{kt}$$

$$\frac{dT}{dt} = k(T-A)$$

i.e. $T = A + ce^{kt}$ is a soln to $\frac{dT}{dt} = k(T-A)$. 2

(ii) T increases from 5 to 15 when $t = 30$ $A = 40$.

at $t = 0$ $T = 5$

$$5 = 40 + ce^{0}$$

$$5 = 40 + c \Rightarrow c = -35.$$

$$\therefore T = 40 - 35e^{kt}$$

when $t = 30$ $T = 15$

$$15 = 40 - 35e^{k \times 30}$$

$$25 = 35e^{30k}$$

$$\frac{5}{7} = e^{30k}$$ 3

8 marks $\ln\frac{5}{7} = 30k \Rightarrow k = \frac{1}{30}\ln\left(\frac{5}{7}\right)$

α) Temperature after further 30 minutes.

$$T = 40 - 35e^{60k}.$$

$$T = 40 - 35.(e^{30k})^2$$

$$T = 40 - 35\left(\frac{5}{7}\right)^2$$ 1

$$\underline{T = 22^\circ}$$

β) Require time to reach 30°c.

$$\therefore 30 = 40 - 35e^{t \times \left(\frac{1}{30}\right)\ln\left(\frac{5}{7}\right)}$$

$$\frac{10}{35} = e^{t \times \frac{1}{30}\ln\left(\frac{5}{7}\right)}$$

$$\ln\frac{10}{35} = t \times \frac{1}{30}\ln\left(\frac{5}{7}\right)$$ 1

$$t = \frac{30\ln\left(\frac{10}{35}\right)}{\ln\left(\frac{5}{7}\right)} \Rightarrow t = \underline{\underline{112 \text{ minutes}}}$$

γ) as $t \to$ large $35e^{kt} \to 0$ as $k < 0$ 1

$\therefore T \to 40$